1 Method of moving frame for intrinsic equivalence problem

Let M^n be a smooth manifold and G be a Lie subgroup.

Definition 1 *A G-structure on M is a reduction of the bundle of frame to a sub-bundle with the structure group G.*

Example 1

- *$O(n)$-structure is a Riemannian structure.*
- *$Gl(m, \mathbb{C})$-structure is an almost complex structure.*
- *$Gl^+(n, \mathbb{R})$-structure is an orientation.*

Let M and $\tilde{M}$ be manifolds of dimension n with G-structure. The equivalence problem is deciding whether there exists a structure-preserving mapping $f : M \mapsto \tilde{M}$. Locally, this is a question of existence of solutions for an overdetermined system of partial differential equations of 1st order.

1.1 Cartan's method for the equivalence problem of G-structure.

Let $\theta = \begin{bmatrix} \theta^1 \\ \theta^2 \\ \vdots \\ \theta^n \end{bmatrix}$ and $\tilde{\theta} = \begin{bmatrix} \tilde{\theta}^1 \\ \tilde{\theta}^2 \\ \vdots \\ \tilde{\theta}^n \end{bmatrix}$ be the fixed frames over M and $\tilde{M}$ respectively, adapted to the G-structure.

Question : when does there exist $f : M \mapsto \tilde{M}$ satisfying $f^* \tilde{\theta}^j = \Sigma_{i=1}^n a_i^j(x) \theta^i$ where $[a_i^j]$ is a G-valued function on M.